

# Optical Rogue Waves in integrable turbulence

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We report optical fiber experiments allowing to investigate integrable turbulence in the focusing regime of the one dimensional nonlinear Schrödinger equation (1D-NLSE). Our experiments are very similar in their principle to water tank experiments with random initial conditions (see M. Onorato *et al.* Phys. Rev. E **70** 067302 (2004)). Using an original optical sampling setup, we measure precisely the probability density function (PDF) of optical power of partially coherent waves rapidly fluctuating with time. The PDF is found to evolve from the normal law to a strong heavy-tailed distribution, thus revealing the formation of rogue waves in integrable turbulence. Numerical simulations of 1D-NLSE with stochastic initial conditions reproduce quantitatively the experiments. Our investigations suggest that the statistical features experimentally observed rely on the stochastic generation of coherent analytic solutions of 1D-NLSE such as Peregrine solitons.

The field of nonlinear optics has recently grown as a favorable laboratory to investigate both statistical properties of nonlinear random waves and hydrodynamic-like phenomena [1–3]. In particular, several recent works point out analogies between hydrodynamics and nonlinear fiber optics in the observation of supercontinuum generation [4], undular bores [2], optical turbulence, laminar-turbulent transition [1] or oceanographic rogue waves (RW) [5, 6].

Rogue waves (RW), also called freak waves, are extremely large amplitude waves occurring more frequently than expected from the normal law [7, 8]. Since the pioneering work of Solli *et al* in 2007 [5], optical RW have been studied in various contexts such as supercontinuum generation in fibers [5, 9–11], optical cavities [12], semiconductor lasers [13], mode-locked fiber lasers [14], laser filamentation [15] and Raman fiber amplifiers or lasers [16, 17].

As stressed out in the recent review [6], there is no obvious analogy between most of the optical experiments on extreme events and oceanography. However a direct correspondence between nonlinear optics and hydrodynamics is provided by the one-dimensional nonlinear Schrödinger equation (1D-NLSE) (see Eq. (1)) that describes various wave systems [6, 18]. In particular, the focusing 1D-NLSE describes at leading order the physics of deep-water wave trains and it plays a central role in the study of RW [6–8, 18, 19].

Modulational instability (MI) is believed to be a fundamental mechanism for the formation of RW [8, 20]. Moreover, analytical solutions of the integrable 1D-NLSE such as Akhmediev breathers (AB), Peregrine solitons or Kuznetsov-Ma solitons (KMs) are now considered as possible prototypes of RW [19, 21–23]. These coherent structures have been generated from very specific, carefully-designed *coherent* initial conditions in optical fiber experiments [21, 23, 24].

On the contrary, oceanic RW emerge from the interplay of *incoherent waves* in turbulent systems. The occurrence of RW in wave turbulence has been theoretically studied in optics [25, 26] and in hydrodynamics [27]. In hydro-

dynamical experiments made with one-dimensional water tanks, non gaussian statistics of the wave height has been found to emerge from random initial conditions [20, 28].

The appropriate theoretical framework combining a statistical approach of *random waves* together with the 1D-NLSE is *integrable turbulence*. This emerging fundamental field of research recently introduced by Zakharov relies on the analysis of complex phenomena found in nonlinear random waves systems described by an integrable equation [29–33]. The mechanisms found in integrable turbulence are of a profoundly different nature than those found in conventional turbulence [3, 29, 34]. A very recent study of the formation of RW in integrable turbulence surprisingly shows that MI leads to stationary statistical state that is not characterized by a high occurrence probability of extreme events [32].

Nonlinear fiber optics is a promising field for the investigation of integrable turbulence because optical tabletop “model experiments” precisely described by the 1D-NLSE can be performed [21, 23, 24, 33]. For instance, the statistical properties of *slowly* fluctuating random waves in *defocusing* integrable turbulence have been recently studied [33]. As MI easily broadens optical spectrum beyond the bandwidth of standard photodetectors, the experimental study of the *focusing* integrable turbulence is very challenging. Despite the numerous works devoted to optical RW, *the generation of extreme events from purely stochastic initial conditions in focusing 1D-NLSE model experiments* remains a crucial and open question [6, 19, 32, 35].

In this letter, we address the fundamental problem of the propagation of random waves described by the focusing 1D-NLSE [30, 32, 35, 36]. We implement an optical fiber experiment conceptually analogous to the water tank experiment described in [20]. Using an original setup to overcome bandwidth limitations of usual detectors, we evidence strong distortion of the statistics of nonlinear random light characterizing the occurrence of optical rogue waves in integrable turbulence.

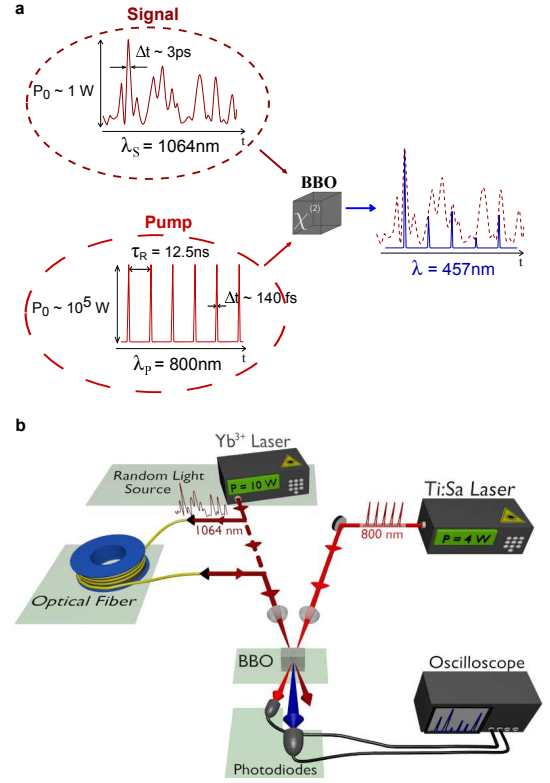
Response times of conventional detectors are usually slower than the typical time scale characterizing power fluctuations of incoherent optical waves. Since the work of Solli *et al.* spectral filters are therefore often used to reveal extreme events in time-domain experiments [5, 10, 17]. In addition to these filtering techniques, shot-to-shot spectrum fluctuations can be evidenced with a dispersive Fourier transform measurement in experiments of pulsed supercontinuum generation [5, 37–39]. Measuring in an accurate and a well-calibrated way the probability density function (PDF) characterizing temporal fluctuations of the power of *random light* is still a challenging task in the field of nonlinear statistical optics.

In order to break through currently-existing detection limitations, we have developed an original setup which allows the precise measurement of statistics of random light rapidly fluctuating with time. Inspired by the time-resolved fluorescence upconversion experiments [40] and by the optical sampling (OS) oscilloscope [41], the principle of our method is based on asynchronous OS (see Fig. 1.a).

Our experimental setup is schematically shown in Fig. 1.b. A “continuous” wave (cw) Ytterbium fiber laser (IPG-YLR series) emitting a linearly polarized partially coherent wave at  $\lambda_S = 1064\text{nm}$  is used as a random light source. This cw laser emits numerous (typically  $10^4$ ) uncorrelated longitudinal modes. The partially coherent wave under investigation is called the “signal”. Blue pulses are generated at a wavelength  $\lambda = 457\text{nm}$  by sum-frequency generation (SFG) between the signal at  $\lambda_S = 1064\text{nm}$  and short “pump” pulses having a central wavelength  $\lambda_P = 800\text{nm}$ . SFG is achieved in a  $5 \times 5 \times 8\text{mm}$  BBO crystal. Non collinear Type I phase matching is achieved with an external angle of  $10^\circ$  between the pump and the signal.

The 140fs-long pump pulses are emitted by a mode-locked Ti:Sa laser (Coherent Cameleon ultra II) with a repetition rate of 80MHz. The maximum output power of the fiber laser is much weaker than the peak power ( $\simeq 4.10^5\text{W}$ ) of the pump pulses. The pump pulses remain therefore undepleted and the peak powers of SFG pulses are proportional to the instantaneous optical powers  $P = P_{(\lambda_S=1064\text{nm})}$  carried by the signal [42]. The variations of SFG pulses power (solid red line in Fig. 2.a) can be seen as snapshots of the fluctuations of the optical power  $P$  carried by the signal. We compute the PDF of  $P$  from the statistical distribution of the peak powers of SFG pulses (red line in Fig 2.b).

The short blue pulses are observed by using a highly sensitive photodiode (MenloSystem FPD310-FV) having a gain of  $\simeq 10^4$  and a rise time of 0.7ns. We record the output of the photodiode with a fast oscilloscope (LeCroy WaveRunner 104MXi-A, bandwidth 1GHz, 10GS/s). We have carefully used the photodiode in a linear regime without any saturation effect. The peak voltage is



**Figure 1. Measurement of the statistics of random light**  
**a. Principle.** The optical sampling of the partially-coherent wave fluctuating with time (the signal) is achieved from sum-frequency generation (SFG). Blue pulses are generated at  $\lambda = 457\text{nm}$  from the interaction of the signal with femtosecond pump pulses inside a  $\chi^{(2)}$  crystal. Each SFG blue pulse has a peak power proportionnal to the initial signal power.  
**b. Setup** The 140fs pump pulses are emitted by mode-locked laser at  $\lambda_P = 800\text{nm}$ . The partially coherent wave is emitted by a Ytterbium fiber laser at  $\lambda_S = 1064\text{nm}$ . We measure statistics of partially-coherent light from the SFG process either directly at the output of the laser or after propagation inside an optical fiber. Pump pulses and SFG pulses are simultaneously recorded with an oscilloscope

proportional to the energy of the corresponding optical pulse. A second photodiode is used to record pump pulses with a high signal-to-noise ratio. This provides a synchronization signal permitting to identify the maxima of SFG pulses. The normalized PDF of the signal is computed from an ensemble of approximately 16 millions measurements of SFG peak powers.

We first measure the PDF at the output of the laser. In all experiments presented in this letter, the mean output power of the Ytterbium laser is fixed at  $\langle P \rangle = 10\text{W}$ . At this operating point, the statistics of the partially coherent wave follows the normal law. Indeed, as plotted in Fig. 2.b, the PDF of the normalized power  $P / \langle P \rangle$  is very close to the exponential function (see Fig. 2.b). It is important to note that the red line on Fig 2.b is not

a fitted exponential function but it represents the exact normalized PDF [ $P / \langle P \rangle$ ] =  $\exp(-P / \langle P \rangle)$ . To the best of our knowledge, PDF of so rapidly fluctuating optical signals has never been *quantitatively* compared to the normalized exponential distribution.

We now use the output of the laser as a random source and we launch the partially coherent signal into an optical fiber in the focusing regime. The fiber is a 15m-long highly nonlinear photonic crystal fiber (provided by Draka France company) having an anomalous dispersion at 1064nm. The fiber maintains the polarization of light and single transverse mode propagation is also achieved. A random light wave with a mean power  $\langle P \rangle = 600\text{mW}$  is launched into the fiber.

Experiments have been carefully designed to be very well described by 1D-NLSE. In particular, the signal wavelength  $\lambda_s = 1064\text{nm}$  is far from the zero-dispersion wavelength  $\lambda_0 \simeq 970\text{nm}$ . Moreover the optical spectral widths (see Fig. 2.c) remain sufficiently narrow to neglect stimulated Raman scattering (SRS) and high-order dispersion effects. The linear losses experienced by optical fields in single pass in the fibers are very negligible. These total losses are around 0.3% in the fiber (loss coefficient of 8dB/km).

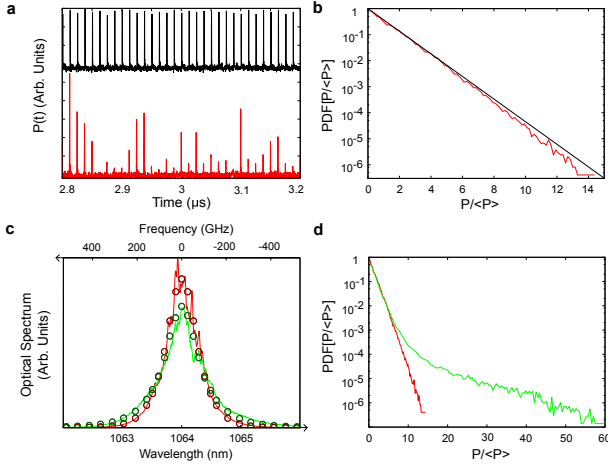


Figure 2. **Experiments** **a.** Pump pulses (black line) and samples of fiber laser fluctuations (SFG pulses, red line). **b.** Statistics of fluctuations at the output of the fiber laser. Probability Density Function (PDF) of normalized optical power  $P / \langle P \rangle$  (red line). The PDF is computed from SFG peak powers plotted in Fig. 2.a. Normalized exponential distribution PDF [ $P / \langle P \rangle$ ] =  $\exp(-P / \langle P \rangle)$  (black line) **c. and d.** **Focusing propagation** Optical power spectra (c.) and PDF (d.) of the partially coherent wave at the input (red line) and output (green line) of the fiber. Experiments are plotted with solid line (c. and d.) and numerical simulations are plotted with circle (c.).

The measurement of the statistics of the optical power after propagation of the partially coherent field in the fiber reveals the occurrence of numerous extreme events

(RW). The comparison between the initial PDF (see red line in Fig 2.d) and the output PDF (see green curve in Fig. 2.d) shows an impressive change in the statistical distribution of optical power. The initial field follows the normal law and its PDF is an exponential function whereas the output PDF of optical power exhibits a strong heavy-tail. The probability of occurrence of very high powers fluctuations (more than 10 times greater than the mean power) is much larger than the probability defined by the normal law. As an example, a fluctuation with a power greater than fifty times the mean power almost never occurs in the initial random gaussian field (one intense fluctuation every  $10^{10}$  seconds) whereas it occurs every  $10^{-6}$  second at the output of the nonlinear fiber.

We have performed numerical simulations of the 1D-NLSE

$$i \frac{\partial \psi}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 \psi}{\partial t^2} - \gamma |\psi|^2 \psi \quad (1)$$

with parameters corresponding to the experiments. At  $\lambda_s = 1064\text{nm}$  the group velocity dispersion coefficient of the fiber is  $\beta_2 = -20\text{ps}^2/\text{km}$ . The effective Kerr coefficient is  $\gamma = 50\text{W}^{-1}\text{km}^{-1}$ .

Numerical simulations are performed by discretizing a temporal window of 618 ps with a set of 8192 points and by using a pseudo spectral splitstep-based method. Mean optical power spectra and PDFs are computed from Monte Carlo simulations performed over an ensemble of 4000 realizations of the initial random process. For each realization of the random initial condition, the initial field is computed the Fourier space with the random phase procedure ([34, 43, 44]) :  $\psi(\omega) = \sqrt{n(\omega)} \exp(i\phi_\omega)$  where  $\phi_\omega$  is a white delta-correlated random process and  $n(\omega) = n_0 \text{sech}(\omega/\Delta\omega)$  is the initial optical spectrum. The width  $\Delta\omega = 2\pi \times 63\text{GHz}$  of the power spectrum of the wave used as initial condition is obtained from a fit of power spectra experimentally recorded.

Optical spectra (see circles in Fig.2.c) and PDFs (see solid green line in Fig. 3.a) computed from the numerical integration of the 1D-NLSE are in quantitative and remarkable agreement with experiments. The comparison between experiments and simulations prove that statistical distributions found in experiments are very well described by the integrable 1D-NLSE.

Numerical simulations of 1D-NLSE allow us to explore the statistics of the wave system at longer propagation distances. This is not feasible in the experiment because other effects such as stimulated Raman scattering play a non-negligible role and strongly break the integrability of the wave system. With the parameters corresponding to our experiments, numerical simulations show that the spectrum and the PDF reach a stationary state for a fiber

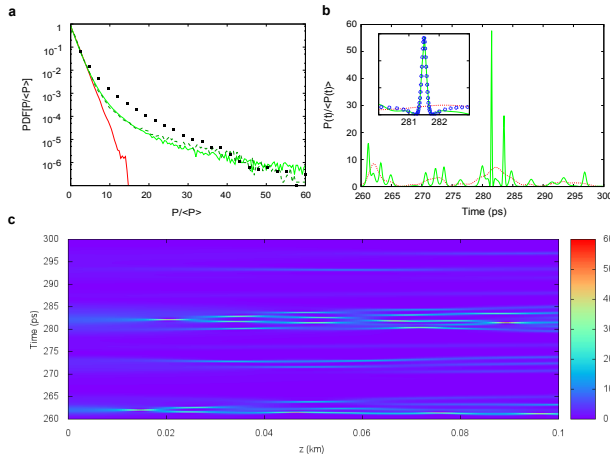


Figure 3. **Numerical simulations.** **a.** PDF of optical power at a propagation length  $z = 0m$  (solid red line),  $z = 15m$  (solid green line) and  $z = 100m$  (stationary state, black squares). Experimental PDF ( $z = 15m$ ) of Fig. 2.d is displayed for comparison (dashed green line). **b.** and **c.** Typical temporal optical power fluctuations along the nonlinear propagations. Green line in c. : fluctuations at  $z = 90m$ . Inset in b. : enlarged view of an intense power fluctuation superimposed on the profile of a Peregrine soliton (blue circle) determined from a best fit procedure.

length  $L \simeq 100m$ . The stationary state of integrable turbulence is here characterized by a strongly heavy-tailed PDF (see black squares in Fig 3.a).

In our experiments, the frequency at which the gain of the MI process is maximum (275GHz) lies within the spectrum of the initial incoherent wave (see Fig. 2.c). On the contrary the author of ref. [32] study numerically the nonlinear stages of the MI of the condensate. In this case the stationary PDF of the wave amplitude is a Rayleigh distribution, which means that the probability of RW formation is the same as in a linear wave system. It is not surprising in integrable systems that the nature of the stationary state depends on the initial conditions. However further theoretical investigations are needed to fully characterize this remarkable difference between two stationary states of integrable turbulence.

Numerical simulations give some insight into the mechanisms underlying the breakdown of the gaussian statistics observed in our experiments. In the focusing propagation, solitons on finite background such as AB, Peregrine solitons or KMs having a short duration and a high power seem to emerge on the top of the highest fluctuations (see Fig. 3.b). The inset of Fig. 3.b evidences an example of the close similarity between the shape of an intense and isolated power fluctuation and the analytical form of a Peregrine soliton.

The results presented in this letter do not only provide fundamental results on rogue waves in integrable turbulence but they also demonstrate a powerful method

to study the statistics of optical fields rapidly fluctuating with time. Indeed the good agreement between numerical simulations and experiments confirms the ability of our apparatus to measure *accurately* the statistics of partially coherent waves with broad spectra. We have estimated from annex pulsed experiments that temporal fluctuations as short as 250fs can be precisely measured with our setup (see further publication). Our method enables for example statistical study of fiber lasers which are remarkable systems for the investigation of optical turbulence [1].

Note that deviations from gaussian statistics have been reported in 1D “spatial experiments” in which the transverse intensity profile of optical beams randomly fluctuating with space is easily recorded by using cameras [8, 45]. In spatial experiments, the speckle fields are localized [45, 46] and the dynamics of random waves is *confined* in a way comparable to the one found in pulsed experiments [47]. Our experiments with “non-decaying” continuous waves widen the perspective by providing new information about nonlinear interactions among *unbounded* random waves in integrable turbulence.

Our experiment made in the anomalous dispersion regime is qualitatively comparable to a one-dimensional deep-water-wave experiment. Starting from random initial conditions, these hydrodynamical experiments have demonstrated the formation of heavy-tailed statistics [20, 28]. However, in water-wave experiments, the extreme events are far less frequent because the height of the waves is strongly limited by the phenomenon of wave breaking [48]. In hydrodynamics, relatively small deviations from gaussianity have been observed and interpreted in the framework of wave turbulence theory [8, 27, 49]. On the contrary, our optical fiber setup provides an accurate laboratory for the exploration of strongly nonlinear random wave systems ruled by the 1D-NLSE.

In this letter, we have quantified how the statistics of stochastic nonlinear optical fields strongly deviate from the normal law in an optical fiber experiment. Our experiments are very well described by the integrable focusing 1D-NLSE and they prove that RW can appear in integrable turbulence [30, 32, 36]. Our experimental and numerical results strengthen the idea that the emergence of deterministic solutions of 1D-NLSE such as AB, Peregrine solitons or KMs in nonlinear random fields is a major mechanism for the formation of rogue waves [19, 50]. From our work, we hope to stimulate further theoretical and experimental investigations permitting to establish a clear connection between the emergence of coherent structures and the statistical properties of nonlinear stochastic fields.

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